Name ______________________________

Section ______________ Date ______________

exercise

42

expansion of the Universe

1 materials Millimeter rule.

2 purpose By measuring the Doppler shift of absorption lines in the spectra of galaxies, their velocities of recession may be determined and the expansion of the universe shown. Both classical and relativistic Doppler formulas are examined and the velocity-distance relation is introduced. The age of the universe is calculated from Hubble's constant.

3 measurement of red shifts Examine Figure 42–1, which shows the spectra of five galaxies. Each spectrum is bounded above and below by a bright line comparison spectrum. Notice that the calcium II lines, H and K, do not appear where they should in each spectrum. In fact they are shifted toward the red, and the further away the galaxy is the further the lines are shifted.

One explanation of this is that galaxies are moving away from us, the universe is expanding. and the spectral lines are shifted by the Doppler effect toward the red end of the spectrum. If that is the case, we can calculate the radial velocity v_r of each galaxy by measuring the amount by which the wavelengths are changed. The radial velocity is given by classical Doppler formula

$$v_r = c\frac{\Delta\lambda}{\lambda} \qquad (42\text{–}1)$$

where c is the speed of light, 3×10^5 km. sec^{-1}, λ is the laboratory wavelength of a spectral line in angstrom units, and $\Delta\lambda$ is the shift of the spectral line in angstrom units.

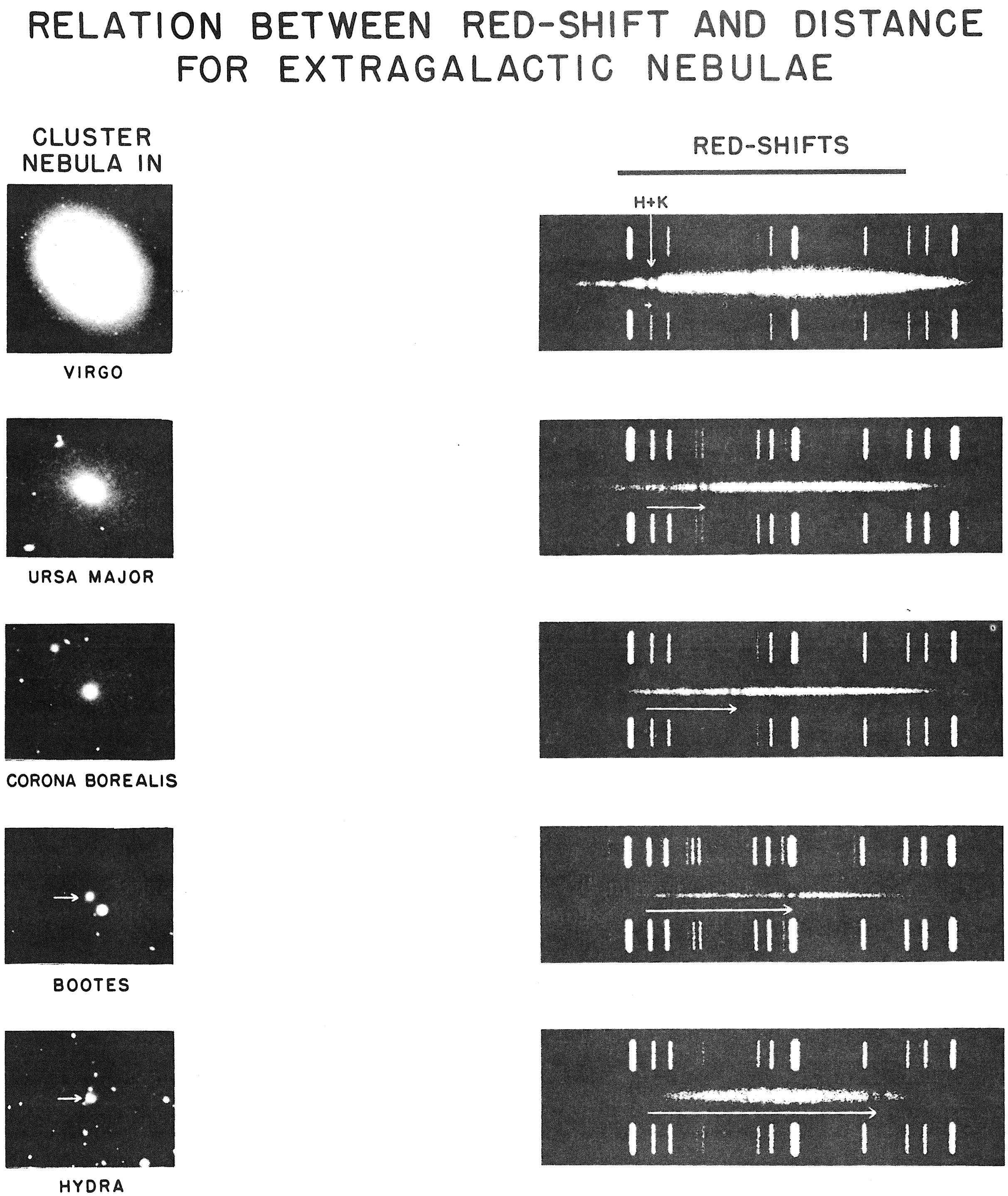

Figure 42-1. Photographs and spectra of extragalactic nebulae. The white arrow under each spectrum indicates the red shift of the H and K lines of calcium. [Courtesy Hale Observatories.]

activity

1. In Figure 42–1 a white arrow has been added to each spectrum to show us how far the lines have been shifted from their normal position. Let ΔR represent the amount of shift. Measure this shift as accurately as possible and record your measurements in Table 42–1 under ΔR.

Table 42–1 Red Shift Data

Galaxy Cluster	ΔR	$\Delta\lambda$	v_r	*r (Mpc)*
Virgo				24
Ursa Major				310
Cor. Borealis				430
Bootes				770
Hydra				1200

2. Measure as accurately as possible the length of the line directly under the word "Red-shifts" in Figure 42–1. This line has been drawn to represent a distance of 100 nm on the spectra. Record the length L of this line in millimeters. $L =$ ____________________ mm.

3. The scale S of the spectra is given by

$$S = \frac{100\,\text{nm}}{L\ \text{mm}} = \text{\_\_\_\_\_\_\_\_\_\_}\text{nm mm}^{-1}.$$

4. Multiply ΔR by the scale to obtain $\Delta\lambda$ in angstroms. If the laboratory wavelength is known, the radial velocity can be calculated. The wavelengths of the two calcium lines are 393.4 nm and 396.8 nm. In this exercise we will use their average wavelength, 395.1 nm, to calcutalte your results in Table 42–1 under v_r.

4 the velocity distance relation

In Figure 42–2, plot each galaxy's velocity versus its distance in megaparsecs (Mpc). Note that there seems to be a linear relation, but it is probably not possible to draw a straight line that passes through every one of the points.

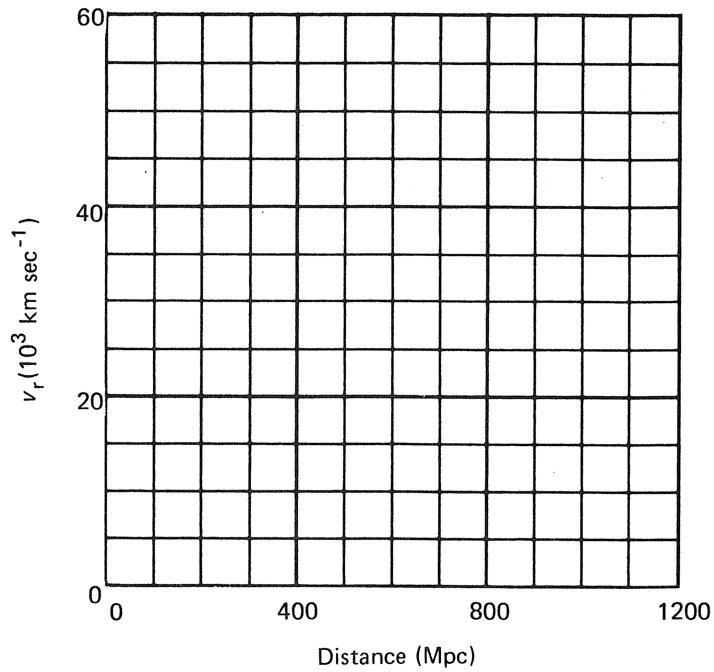

Figure 42-2. Velocity-distance diagram.

activity

1. Draw a straight line through the origin such that the points are as close to the line as possible. Why should this line pass through the origin?

2. The slope of the line you have drawn is the increase in the velocity divided by the increase in distance.

   (a) If the distance increases by 400 Mpc, the velocity will increase by __________ km sec^{-1}.

   (b) The slope is then ____________ km sec^{-1} Mpc^{-1}.

   Note: This is Hubble's constant and is approximately 50 km sec^{-1} Mpc^{-1}.

problems

1. We have discovered a cluster of galaxies in which the H and K lines of calcium are shifted by 430 nm.

(a) What is your estimate of the distance to this cluster? ____________

(b) Upon what assumptions does your estimate depend?

2. In the quasi-stellar object PHL 1127 the Lyman alpha line of hydrogen, which should be at 121.6 nm is detected near 364.8 nm.

(a) Calculate the radial velocity of this object using the classical Doppler formula, equation (42–1).

$v_r =$ ____________ km sec^{-1}

(b) Is this reasonable? Why?

(c) Calculate the velocity using the relativistic equation

$$\frac{v_r}{c} = \frac{(1+Z)^2 - 1}{(1+Z)^2 + 1}$$

where $Z = \frac{\Delta\lambda}{\lambda}$

$v_r =$ ____________ km sec^{-1}

(d) When velocities are comparable with the speed of light, the relativistic form of the equation must be used. But when the velocities are low, say less than 10% of the speed of light, the simpler form may be used. Both forms must give nearly the same answer at low velocities. Calculate the radial velocity of the galaxy in the Hydra custer (Table 42–1) using the relativistic form of the equation.

$v_r =$ ________ km sec^{-1}

(e) Does the value from part (d) agree with the radial velocity you obtain ed with the simpler form?

3. Hubble's constant, H, has the units

$$\frac{\text{km}}{(\text{sec})\ (10^6\ \text{pc})}.$$

If the formula $v_r = Hr$ is expressed in the form $r/v_r = 1/H$ it is clear that $1/H$ represents time. The units for $1/H$ are

$$\frac{(\text{sec})\ (10^6\ \text{pc})}{\text{km}}$$

Now $1/H$ is the approximate age of the Universe (the time since the last big bang). Calculate the age of the Universe in years using

$$H = 50 \frac{\text{km}}{(\text{sec})\ (10^6\ \text{pc})}.$$

Hint: There are 3×10^7 sec in a year and 1 pc $= 3 + 10^{13}$ km approximately.

Age of the Universe ________ years

supplementary problems

1. Show that the formulas

$$\frac{v_r}{c} = \frac{(1+Z)^2 - 1}{(1+Z)^2 + 1}$$

and

$$\frac{\Delta\lambda}{\lambda} = \frac{1 + \dfrac{v_r}{c}}{\sqrt{1 - \left(\dfrac{v_r}{c}\right)^2}} - 1$$

are equivalent.

Note that $Z = \dfrac{\Delta\lambda}{\lambda}$.

2. Show that when v_r is very much less than c, ($v_r << c$), the classical Doppler formula

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$

may be derived from the relativistic formula. *Hint*: If $v_r << c$, what can you say about $\left(\frac{v_r}{c}\right)^2$?

3. If $\Delta\lambda/\lambda = 0.1$, show that the error in v_r (as compared with the relativistic value) is about 5% if you use the classical Doppler formula.